# Partitioning Colored Points into Monochromatic Islands is NP-Complete

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## Abstract

We are given a set $S$ of colored points and a positive integer $\kappa$. A subset of $S$ is monochromatic if it contains points of only one color. We prove that it is NP-complete to decide whether $S$ can be partitioned into at most $\kappa$ monochromatic subsets whose convex hulls are pairwise-disjoint.

## 1 Introduction

Research on partitioning and separating colored points is abundant in both discrete and computational geometry. For an overview of discrete-geometry results, we refer to a recent survey [19]. The shapes used to partition or separate the points may have constant-complexity such as lines [8, 10, 15, 16, 18, 21, 22], triangles [2, 7, 23], disks [9, 11, 24], rectangles [1, 5, 29], wedges [15, 17], strips [15, 17], or L-shapes [25]; or they may have linear complexity such as convex polygons [3, 4, 12, 13].

Here we study the problem of using convex polygons to partition colored points into monochromatic islands. A set of points is *monochromatic* if it contains points of only one color. For a colored point set $P$, an *island* $I$ is defined by Bautista-Santiago et al. [4] to be a subset of $P$ such that $\mathcal{CH}(I) \cap P = I$, where $\mathcal{CH}(I)$ denotes the convex hull of $I$ (including the interior).

We prove that the following problem is NP-complete (see Figure 1 for an illustration of the problem):

**2C-IP** Given a bichromatic set of points and a positive integer $\kappa$, does a partition of the points into at most $\kappa$ monochromatic islands exist?

We identify an island (a subset of $P$) with its convex hull (a subset of $\mathbb{R}^2$). In particular, a partition into islands has the requirement that the convex hulls of the islands are pairwise-disjoint. Furthermore, we will refer to the two colors in the 2C-IP problem as red and blue.

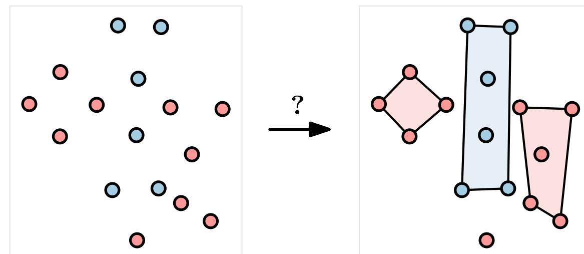


Figure 1: The 2C-IP problem. A set of red and blue points and a partition into four monochromatic islands.

**Related work.** Monochromatic island partitions have been studied before [12, 27]. There is also work on finding an optimal monochromatic island in a colored point set [4] (with a flexible definition of 'optimal'), and island partitions have also been used to define a notion of *coarseness* that captures how blended a set of bicolored points are [6]. However, to our knowledge, the computational complexity of the 2C-IP problem has not been studied. The problem of deciding whether a bichromatic set of points can be *covered* by $\kappa$ monochromatic islands is known to be NP-complete [4] as it follows from a reduction by Agarwal and Suri [2]. Agarwal and Suri reduce from the planar 3-SAT problem by using one color of points to constrain islands of the other color. However, this reduction fails for the 2C-IP problem: islands formed by blocker points cannot overlap islands of the other color. This fact makes an NP-hardness reduction more challenging. We circumvent this issue in our reduction by creating a structured problem instance where the optimal partitions of the blue points and the red points have little interaction. Our reduction is inspired by the reduction by Van Kreveld, Speckmann, and Urhausen [28] for a similar problem.

## 2 Overview

We show hardness via a reduction from an independent-set problem on line segments (which we will simply refer to as segments from now on). The problem asks: given a set of $n$ segments and a positive integer $k$, does a subset of $k$ segments exist that are pairwise disjoint? Kratochvil and Nešetřil [20] showed this to be NP-complete, even if the segments are aligned with exactly three directions and no segment endpoint lies on another segment. We adapt their reduction to show that this problem is

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still NP-complete even if there exists a regular triangular grid $T$ defined by $O(n^2)$ grid lines such that:

- the segments and their endpoints lie on grid lines and grid vertices of $T$ respectively;
- no pair of segments lie on the same grid line of $T$.

Following Kratochvil and Nešetřil's notation we refer to this problem as pure (segments cannot contain other endpoints) equiangular 3-directional segment maximum independent set (PEA3D) and show it is NP-complete in Section 3. While it is not surprising that the problem stays NP-complete under the given restrictions and the proof requires only minor changes to the original reduction [20], this variant might be of independent interest as a basis for reductions to problems on triangular grids.

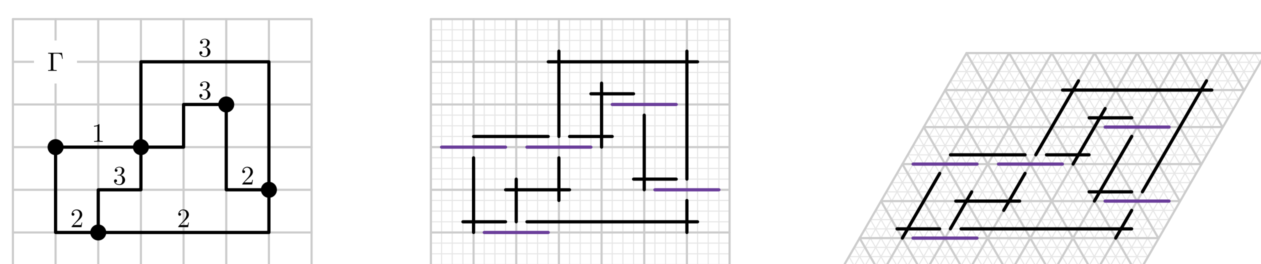
In Section 4 we reduce PEA3D to 2C-IP. The triangular grid $T$ of the PEA3D instance is used as a basis for the 2C-IP instance. We shrink each face of $T$ slightly, which gives rise to narrow corridors in which we place blue points to represent the segments of PEA3D. We also place blue points in the interior of the triangles and red points on the boundary of triangles. Our 2C-IP instance separates red and blue island candidates and limits their interaction in the following sense. We show that there exists an optimal partition $P_r$ of the red points into monochromatic islands (avoiding the blue points but not blue islands) and an optimal partition $P_b$ of the blue points into monochromatic islands (avoiding the red points), such that the islands in $P_r$ and $P_b$ are pairwise disjoint. Our instance provides this separation by allowing us to place blue points in the interior of triangles to block red islands, without needing additional islands to cover these 'blockers'. Similarly, red blockers can be freely placed on the boundary of triangles.

## 3 NP-completeness of PEA3D

Kratochvil and Nešetřil [20] reduce the independent-set problem in a planar graph $G = (V, E)$ of maximum degree 4 (which remains NP-hard [14]) to finding an independent set of segments with exactly three distinct slopes. To show hardness of PEA3D we retrace the steps of this reduction and make small changes in several steps to guarantee that the constructed set of segments complies with our additional restrictions.

Their reduction starts by creating a planar orthogonal drawing $\Gamma$ of $G$ (in polynomial time and on an $O(|V|) \times O(|V|)$ grid [26]; an example is shown in Figure 2a). Every edge $e \in E$ is represented by a polyline with $k_e$ linear pieces (the numbers in Figure 2a). The idea of the reduction is to represent $\Gamma$ by a set $S$ of segments, such that the maximum independent-set size of $S$ is proportional to the maximum independent-set size of $G$. The set $S$ of segments Kratochvil and Nešetřil create form an intersection representation of an auxiliary graph $G'$; that is, there is a segment in $S$ for every vertex of $G'$ and two segments intersect if and only if their vertices are connected by an edge. Graph $G'$ is constructed by subdividing edges of $G$. To recreate drawing $\Gamma$ with segments, every edge of $G$ needs to be subdivided by at least $k_e$ vertices, and to ensure the relation between independent-set sizes every edge of $G$ needs to be subdivided by an even number of vertices. Hence, Kratochvil and Nešetřil define $G'$ by subdividing each edge by adding an even number $2\lfloor(k_e+1)/2\rfloor+2$[1] of vertices. This implies the size $\alpha(G')$ of a maximum independent set in $G'$ is equal to $\alpha(G) + \sum_{e \in E}(\lfloor(k_e+1)/2\rfloor + 1)$. They then create $S$ by replacing every vertex of $\Gamma$ with a horizontal segment and replacing every linear piece of an edge in $\Gamma$ with a corresponding horizontal or vertical segment (Figure 2b). These edge segments are slightly elongated to create proper intersections at the points where edges in $\Gamma$ have bends except for the segments that would intersect the vertex segments. These are instead slightly shortened on that end and if they are horizontal segments they are additionally slightly vertically offset to ensure that the vertex segments are not inter-

[1]The original publication typesets this as $2\,[(k_e+1)/2]+2$.




(a) Drawing $\Gamma$; numbers show the complexity $k_e$ of an edge $e$

(b) Segment representations of vertices (purple) and linear pieces of edges (black)

(c) Segments after applying the scaling and skewing transformations

Figure 2: The first steps of the hardness reduction for PEA3D. We assume for illustration purposes that vertices in $\Gamma$ do not lie directly next to each other, so that the segments representing vertices can be wide. This can always be ensured by doubling the resolution of the grid.

sected. Note that these operations can be performed such that the segments still lie on a regular square grid by doubling the resolution of the grid a constant number of times. So far, $k_e$ segments have been placed per edge $e$. Before the next step of the reduction we apply our first adaption, which is applying the following linear transformation to $S$:

$$\begin{pmatrix}1 & \tan(\pi/6)\\ 0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 0 & \sqrt{3}/2\end{pmatrix}.$$

This keeps horizontal segments horizontal, and first shrinks and then skews the vertical segments to ones of the same length but now with a slope of $\pi/3$ (this is shown in Figure 2c). To complete the segment intersection representation of $G'$ (Figure 3a), Kratochvil and Nešetřil add an additional 2 or 3 segments if an edge $e$ in $\Gamma$ consists of an even or odd number of linear segments respectively. If $e$ has an even number of segments, we place on both ends of $e$ a small segment with slope $-\pi/3$ such that it intersects $e$ and the vertex segment (case I in Figure 3b). If $e$ has an odd number of segments, we similarly add on one end a single segment with slope $-\pi/3$; however, on the other end we instead add two segments with slopes $-\pi/3$ and $\pi/3$ (case II in Figure 3b). Note that these connection segments can always be placed in this way because the vertices are represented by segments with slope 0, and linear pieces of edges by segments with slope either 0 or $\pi/3$. Further note that it suffices to double the regular triangular grid a constant number of times such that the connection segments can be placed on the grid.

This ends the reduction by Kratochvil and Nešetřil; however, we now apply our final adaption. Let $n$ denote the number of segments in $S$. There might still be up to $O(n)$ segments that lie on the same grid line. To avoid this, we can simply double the resolution of the grid until every grid line is replaced by sufficiently many (at most $O(n)$) grid lines, i.e., as many grid lines as there are line segments lying on that specific grid line (Figure 3c). It is easy to see that (i) any two segments that lay on the same grid line can be shifted to be on different grid lines in such a way that any two segments that intersected before still intersect after being shifted, and (ii) this procedure adds at most $O(n)$ grid new lines per original grid line and therefore the entire set of segments fits into a regular triangular grid with $O(n^2)$ grid lines.

By construction, the final set of segments forms an intersection representation of $G'$, which has an independent set of size at least $k + \sum_{e\in E}(\lfloor (k_e+1)/2\rfloor + 1)$ if and only if $G$ has an independent set of size at least $k$. Therefore, the PEA3D problem is NP-hard.

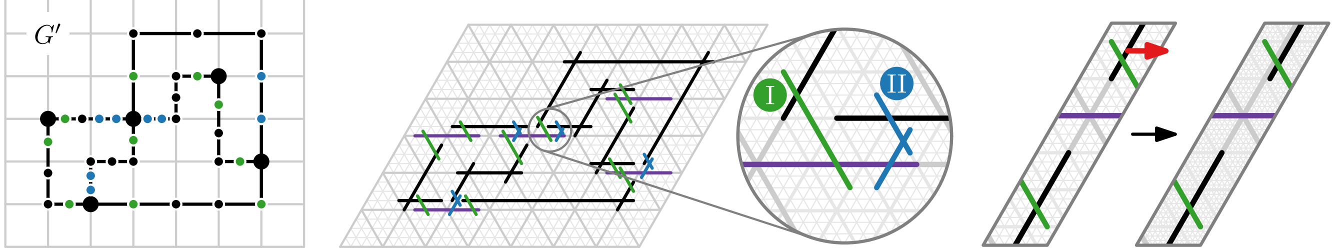
Lastly, NP-containment is straightforward since we can easily represent an independent set of segments embedded in an integer grid in polynomial space, and verify in polynomial time that (i) they are pairwise non-intersecting and (ii) that the set has a large enough cardinality. This completes our reduction and results in the following theorem.

**Theorem 1** PEA3D *is* NP*-complete.*

## 4 NP-completeness of 2C-IP

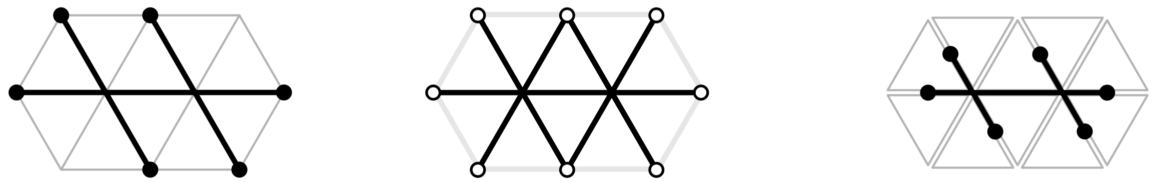
We are now set up to present our reduction from PEA3D to 2C-IP. Let $S$ be a set of segments and let $k$ be a positive integer such that together they form an instance of PEA3D. We will construct a set $P$ of red and blue points and a value $\kappa$ such that there exists an independent set of $S$ of size $k$ if and only if there exists a partition of $P$ into $\kappa$ monochromatic islands.

**Auxiliary structure.** Our reduction uses as auxiliary structure a bounded triangular unit-grid on which the segments $S$ lie. We choose this grid such that it has a convex outer boundary and such that no interior point of a segment lies on this boundary. See Figure 4 for an illustration of this grid and upcoming definitions. Let $T$ denote the set of points in $\mathbb{R}^2$ that lie on the edges and vertices of the grid that are not incident to the outer face. We shrink each (triangular) face of the grid by taking the Minkowski difference with a disk of diameter $\varepsilon = 0.1$ to create a set $\Delta$ of triangles. The




(a) Graph $G'$ (b) Intersection representation of $G'$ (c) Moving segment on new grid line

Figure 3: The last steps of the reduction for PEA3D. The segments in (b) lie on a regular triangular grid consisting of $O(n)$ grid lines. Connections to vertex segments are made using either one (I) or two (II) segments, which correspond to green and blue subdivision vertices of (b), respectively. After step (c), there are $O(n^2)$ grid lines.



(a) Segments $S$ in triangular grid (b) Set $T$ (in black, open at markers) (c) Triangles $\Delta$ and shortened segments

Figure 4: Structure of the 2C-IP instance.

free space between the triangles of $\Delta$ form narrow *corridors* of width $\varepsilon$. Note that each corridor corresponds to a grid line—which we will refer to as *rows* from now on—in $T$. We call the areas where three corridors meet *junctions*. Each junction corresponds to an *interior grid point* in $T$: a point where grid rows intersect. The segments of corridor in between two junctions form *alleys*, which correspond to edges of the grid $T$. For example, grid $T$ in Figure 4 has 5 rows and 2 interior grid points, and the corresponding set of triangles $\Delta$ has 5 corridors, 2 junctions, and 11 alleys. Lastly, we shorten each segment in $S$ on both sides by half a unit such that they end exactly in the middle of a grid edge; note that this preserves the intersection pattern since the endpoints of a segment in $S$ do not lie on any other segment in $S$.

**2C-IP instance.** We create points of $P$ in two phases. The first set of points represent triangles in $\Delta$ or segments in $S$; the second set of points limits the type of monochromatic islands that can connect points of the first set. We begin by placing a blue point at the centroid of every triangle (*triangle points*) and a blue point at the end of every segment (*segment points*). In every alley we place a red point at the center of each of the two triangle edges it is bounded by; we call these *bracket points* and the pair a *bracket*. See Figure 5 for an illustration. If the alley also contains a segment point, we say the bracket is *filled*, otherwise the bracket is *empty*. Each bracket point lies in exactly one corridor and one alley. Note that for each alley, the two bracket points in that alley as well as a potential segment point are all collinear with the triangle points of the triangles in $\Delta$ that bound the alley. This completes the first phase of the point placement. Let $B$ and $R$ be the set of all blue and red points placed so far, respectively.

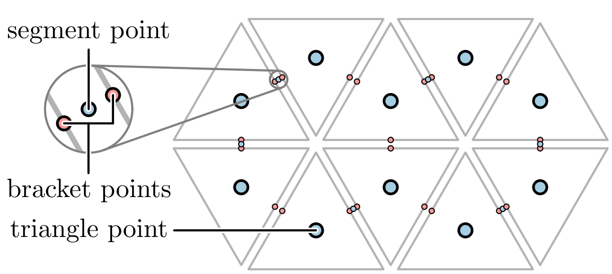



Figure 5: Placement of the first set of points.

In the second phase we place a set of red and a set of blue *blocker* points. For every pair $p, q$ of distinct points in $B$ such that $p$ and $q$ are not segment points of the same segment, the segment $\overline{pq}$ intersects the boundary of a triangle in $\Delta$. This intersection exists since two segment points lie in the same corridor only if they originate from the same segment; indeed, no two segments of $S$ lie in the same grid row, which implies that that no two endpoints of distinct shortened segments lie in the same grid row. We place a red blocker point at an arbitrary such intersection (Figure 6). Any island that contains both $p$ and $q$ also contains the red blocker point. Hence, after placement of these blockers, we have the following property.

**Observation 1** *Two distinct blue points $p, q \in B$ can be part of the same monochromatic island only if $p$ and $q$ are the segment points of the same segment.*

Similarly for every pair $(r, s)$ of red points in $R$ lying in different corridors, the segment $\overline{rs}$ intersects a triangle. Consider the three straight-line segments emanating from the centroid of the triangle and ending at one of the corners of the triangle respectively. The segment $\overline{rs}$ intersects at least one of these straight-line segments. We place a blue blocker at an such arbitrary intersection (Figure 6). This yields a similar property as before, but now for the red points.

**Observation 2** *Two points $r, s \in R$ can be part of the same monochromatic island only if they lie in the same corridor and do not form a bracket that is filled.*

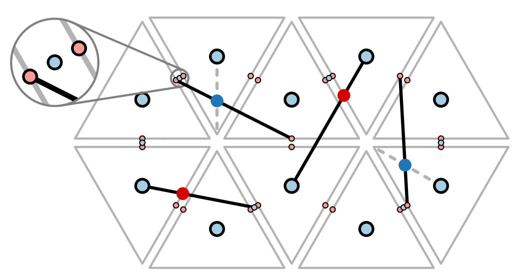


Figure 6: Examples for placement of blockers (solid red and solid blue points).

Note that all red blocker points lie on the boundary of triangles in $\Delta$ and all blue blocker points lie in the interior of triangles in $\Delta$; hence, the sets of red and blue blocker points are disjoint. Let $Z$ be the set of all placed blocker points. The final set of constructed points is $P = B \cup R \cup Z$. Note that Observations 1 and 2 apply only to pairs of points in $B$ and $R$, not between any two blue points or any two red points in $P$. Therefore, an island may, for example, cover blue points in different triangles given at most one of them is a triangle point.

With this the construction of our instance is almost complete. It remains to state the value $\kappa$. Let $\rho$ and $\theta$ be the number of rows and interior grid point in grid $T$ respectively. We set $\kappa = |\Delta| + \rho + 4|S| + 2\theta - 2k$. The reason behind this particular choice for the value of $\kappa$ will become apparent in the following sections.

**Triangular grid.** Before arguing how the constructed instance can be used to solve the independent-set problem, we state two observations on triangular grids.

**Observation 3** *There is a bijection between minimum-cardinality segment partitions of $T$ and maps that choose a grid axis at every interior grid point of $T$.*

Note that the segments in the observation above are not related to the set $S$ of segments of the independent-set problem. Intuitively, a map such as described in the lemma above chooses at every interior grid point the segment of which row 'continues', with the segments from the other rows stopping at the grid point. Such a choice has to be made as the grid point should be covered by exactly one segment; conversely, each choice uniquely defines a minimum-cardinality segment partition (Figure 7). It is both sufficient and necessary to use one segment for each grid row and two additional segments for each interior grid point. Therefore, we state the following observation.

**Observation 4** *A minimum-cardinality partition of $T$ into segments has size $\rho + 2\theta$.*

**NP-completeness.** We now prove that a minimum-cardinality partition of $P$ into monochromatic islands has cardinality $\kappa$ if and only if $k$ is the size of the maximum independent set of $S$. We start by proving the upper bound.

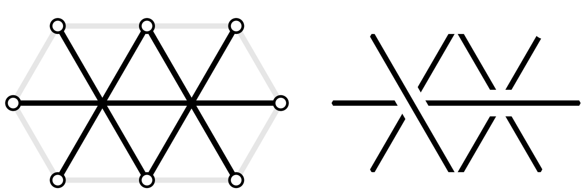


Figure 7: Left: grid $T$ (markers and gray boundary are not part of $T$). Right: segment partition of $T$ of size 9.

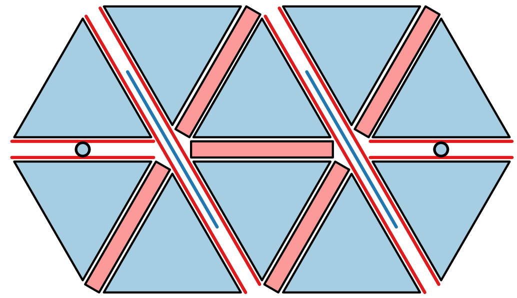


Figure 8: The structure of an optimal island partition. For illustration purposes the corridors have been made wider and the red islands have been moved into the corridors instead of lying on the boundary of triangles.

**Lemma 2** *If $S$ has an independent set of size $k$ then $P$ can be partitioned into $\kappa$ monochromatic islands.*

**Proof.** Let $N$ be the independent set of $S$ of size $k$. A partition of $P$ into $\kappa$ islands can be constructed as follows; see Figure 8 for an example. Cover the interior of each triangle in $\Delta$ with a blue island. For each segment in $N$, cover the corresponding pair of segment points with a blue island. For each segment not in $N$, cover each of the corresponding segment points with a singleton island. To construct the islands covering red points, at every interior grid point of $T$ we choose an axis as follows. If the grid point is covered by a segment in $N$ then we choose the axis with which the segment is aligned; otherwise we choose an arbitrary axis. By Observation 3, this mapping of grid point to axis corresponds to a partition of $T$ into segments. Call this set of segments $L$. For each segment $\ell \in L$ that covers a segment point in $P$, create two corresponding segment-shaped islands parallel to $\ell$ to cover red points in the corresponding alleys. For each segment in $L$ that does not cover a segment point in $P$, create one quadrilateral island to cover the red points. A red point of $P$ may lie exactly on the vertex of a triangle. Such a point would be covered by two islands in our construction; we can fix this simply by deleting such a point from one of the two islands.

All blue points are either segment points or lie in the interior of a triangle of $\Delta$, both of which are covered by islands. All red points lie on the edges of corridors, which are also all covered by islands. Furthermore, all islands are pairwise-disjoint and monochromatic by construction. The problem instance uses $|\Delta|$ islands to cover the interiors of triangles and $2|S| - k$ islands to cover segment points. By Observation 4, set $L$ has size $\rho + 2\theta$. Each of the $k$ segments in $L$ that covers two segment points is split into two red islands. Each of the $2(|S| - k)$ segments in $L$ that covers one segment point is split into two red islands. The remaining segments correspond to exactly one red island. Hence, the total number of islands is:

$$|\Delta| + 2|S| - k + \rho + 2\theta + k + 2(|S| - k)$$
$$= |\Delta| + 4|S| - 2k + \rho + 2\theta = \kappa$$

□

Next, we match the upper bound with a corresponding lower bound.

**Lemma 3** *If $S$ has a maximum independent set of size $k$ then $P$ cannot be partitioned into fewer than $\kappa$ monochromatic islands.*

**Proof.** We argue that any monochromatic island partition of $P$ uses at least $\kappa$ islands. Our argument consists of two parts, one for the set of blue points and one for the set of red points.

The set $B$ has by construction cardinality $|\Delta| + 2|S|$. By Observation 1, a monochromatic island that covers points of $B$ covers either (i) a single point of $B$ or (ii) a pair of segment points belonging to the same segment. Note that each island of type (ii) covers a distinct segment in $S$ in the sense that the points in the convex hull of the island are a superset of the points on the segment. A set of islands of type (ii) then cover a set of pairwise non-intersecting segments of $S$. As such an independent set of $S$ has size at most $k$, there can be at most $k$ islands of type (ii) in an island partition. Hence, an island partition of $P$ requires at least $|B| - k = |\Delta| + 2|S| - k$ blue islands.

By Observation 2, red points in $R$ can be part of the same monochromatic island only if they are in the same corridor; furthermore, red islands from only one row can cross a junction. Therefore, by Observation 3, an island partition can be turned into a segment partition of $T$: at every grid point of $T$, if an island using at least two points in $R$ crosses the corresponding junction, then choose the grid axis that corresponds to the corridor in which the two red points lie; otherwise, choose an arbitrary axis. Call this segment partition $L$. We map a point $r \in R$ to the segment of $L$ that crosses the alley that contains $r$. By Observation 4, this yields $\rho + 2\theta$ sets of red points, each corresponding to a segment in $L$. The convex hull of such a set of points can contain either (i) no segment point, (ii) one segment point, or (iii) two segment points. Note that, by construction, an island of the partition covers points from at most one set. Therefore, we can bound the number of islands by bounding the number of sets. Sets of type (i) require at least one island (as they are non-empty). Sets of type (ii) and (iii) require at least two islands. Sets of type (iii) arise from segments of $L$ that cover a set of pairwise non-intersecting segments of $S$. Hence, as an independent set of $S$ has size at most $k$, there are at most $k$ sets of type (iii). Thus, there are at least $2|S| - 2k$ sets of type (ii) or (iii), and an island partition of $P$ requires at least $\rho + 2\theta + k + (2|S| - 2k) = \rho + 2\theta + 2|S| - k$ red islands.

By summing the two lower bounds, we find that a monochromatic island partition of $P$ uses at least $|\Delta| + 4|S| - 2k + \rho + 2\theta = \kappa$ islands. □

The two lemmas combine to form our main result.

**Theorem 4** 2C-IP *is* NP*-complete.*

**Proof.** Let $(S, k)$ be a instance of PEA3D and $(P, \kappa)$ the corresponding instance of 2C-IP. By Lemma 2 and 3, set $S$ has an independent set of size $k$ if and only if $P$ can be partitioned into $\kappa$ monochromatic islands. Hence, because $(P, \kappa)$ can be constructed in polynomial time from $(S, k)$ and has polynomial size, NP-hardness follows from Theorem 1. To show containment in NP, note that we can represent a solution to 2C-IP in polynomial space as each island is a subset of $P$ and we can check in polynomial time whether a set of subsets of $P$ is indeed a monochromatic island partition of $P$. □

## 5 Conclusion

We have shown NP-completeness of 2C-IP. The 2C-IP problem is related to the similar problem 2C-TP where triangles are used instead of convex polygons to partition the points; variants of 2C-TP have been studied by Agarwal and Suri [2] and Bergold et al. [7]. Our reduction does not apply immediately to the 2C-TP problem, so the complexity of 2C-TP is still open. It does, however, hold when covering with quadrilaterals. Our reduction makes heavy use of collinear points. By replacing every point in $R$ and $B$ by an appropriate pair of points, we suspect the reduction can be adapted to hold true for points in general position.

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